Efficient Target Estimation in Distributed MIMO Radar via the ADMM

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We consider the problem of target estimation in distributed MIMO radars that employ compressive sensing.

We formulate a sparse signal recovery problem with
- magnitude constraints on the target reflection coefficients;
- a special structure for the signal to be recovered consisting of equal size blocks that have the same sparsity profile.

A solution is proposed based on the alternating direction method of multipliers (ADMM), which
- is computational more efficient as compared to algorithms based on the interior point method;
- has improved estimation accuracy resulting from exploiting prior information on the target reflection coefficients;
- is robust over a wide range of a manually chosen parameter.

A parallel implementation and a decentralized scheme are discussed.
We consider a MIMO radar system with $M_t$ transmit nodes (TX) and $M_r$ receive nodes (RX) that are widely separated.

To exploit the spatial sparsity of the targets, the location space is discretized on the grid $\Theta = \{(x_n, y_n), n = 1, \ldots, N\}$.

The received baseband signal at the $j$-th RX $z_{ij}(t)$ arising due to the transmission of the $i$-th TX [Petropulu, Yu & Huang 2011]:

$$z_{ij}(t) = \sum_{n=1}^{N} s_{ij}^n x_i(t - \tau_{ij}^n) + n_{ij}(t) \quad (1)$$

<table>
<thead>
<tr>
<th>$x_i(t)$</th>
<th>The $i$-th waveform</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{ij}^n$</td>
<td>Reflectivity associated with the $n$-th grid point and TX/RX pair $(i,j)$</td>
</tr>
<tr>
<td>$\tau_{ij}^n$</td>
<td>Time delay associated with the $n$-th grid point and TX/RX pair $(i,j)$</td>
</tr>
<tr>
<td>$n_{ij}(t)$</td>
<td>Noise for TX/RX pair $(i,j)$</td>
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</tbody>
</table>

If there is a target located on the $n$-th grid point, then $s_{ij}^n$ is the target complex RCS with $|s_{ij}^n| \in [0, \omega_0] \triangleq \Omega$; otherwise $s_{ij}^n$ is zero.
Obtain $L T_s$-spaced samples and express in vector form

$$z_{ij} = \Psi_{ij} s_{ij} + n_{ij}$$

where $s_{ij} = [s_{ij}^1, \ldots, s_{ij}^N]^T$ and

$$\Psi_{ij} = \begin{bmatrix}
    x_i(t_0 + 0 T_s - \tau_{ij}^1) & \cdots & x_i(t_0 + 0 T_s - \tau_{ij}^N) \\
    \vdots & \ddots & \vdots \\
    x_i(t_0 + (L-1) T_s - \tau_{ij}^1) & \cdots & x_i(t_0 + (L-1) T_s - \tau_{ij}^N)
\end{bmatrix}_{L \times N}$$

The signal model for the overall MIMO radar system is

$$z = \left[ (z_{11})^T, \ldots, (z_{MtMr})^T \right]^T = \Psi s + n$$

where $\Psi = \text{diag}(\Psi_{11}, \ldots, \Psi_{MtMr})$, $s = \left[ (s_{11})^T, \ldots, (s_{MtMr})^T \right]^T$ and

$$n = \left[ (n_{11})^T, \ldots, (n_{MtMr})^T \right]^T.$$

$s$ exhibits group sparsity: it is composed by $MtMr$ sub-blocks, which share the same sparsity profile.
Group sparsity (also known as block sparsity) was exploited to achieve improved target estimation and further reduction of the number of measurements needed.

Existing block sparse recovery methods used for distributed MIMO radars include

- Block Orthogonal Matching Pursuit (BOMP) [Gogineni, Nehorai 2011]: Poor performance in noise
- Group Lasso with proximal gradient algorithm (GLasso) [Petropulu, Yu & Huang 2011]: High complexity, sensitive to the manually tuned parameter
- mixed $\ell_1/\ell_2$ norm optimization (L-OPT) [Li, Petropulu 2014]: High complexity, assuming known noise variance

We are aiming for a recovery method with

- low complexity and robust performance
- flexibility of incorporating prior information
Reformulate for real variables:

\[
\begin{bmatrix}
\Re\{z\} \\
\Im\{z\}
\end{bmatrix} = 
\begin{bmatrix}
\Psi & \tilde{\Psi}
\end{bmatrix}
\begin{bmatrix}
\Re\{s\} \\
\Im\{s\}
\end{bmatrix} + 
\begin{bmatrix}
\Re\{n\} \\
\Im\{n\}
\end{bmatrix}
\]

where \(\tilde{\Psi}\) is still block diagonal, and \(\tilde{s} \in \mathbb{R}^{2N_{M_t}M_r}\) has group sparsity.

Solve the convex optimization problem

\[
\min \frac{1}{2} \|\tilde{z} - \tilde{\Psi}\tilde{s}\|_2^2 + \lambda \sum_{n=1}^{N} \|\tilde{s}[\mathcal{I}_n]\|_2
\]

s.t. \(\tilde{s} \in \Omega^{2N_{M_t}M_r}\)

- The set \(\mathcal{I}_n, \forall n \in \mathbb{N}_N^+\) with cardinality \(2M_t M_r\) indexes out entries in \(\tilde{s}\) corresponding to the \(n\)-th grid point.
- The constraint \(\tilde{s} \in \Omega^{2N_{M_t}M_r}\) is satisfied if \(\|[\tilde{s}[i], \tilde{s}[i + N_{M_t}M_r]]\|_2 \in \Omega, \forall i \in \mathbb{N}_{NM_t M_r}^+\).
In order to use Alternating Direction Method of Multipliers (ADMM), we introduce the auxiliary variables $y$ and $x$. The problem then becomes

$$\min \frac{1}{2} \| \tilde{z} - \tilde{\Psi} \tilde{s} \|_2^2 + \sum_{n=1}^{N} \lambda \| y_n \|_2$$

s.t. $y_n = D_n \tilde{s}, \forall n \in \mathbb{N}_N^+; \quad x = \tilde{s}, \quad x \in \Omega^{2NM_tM_r}$

where the matrix $D_n$ selects the entries of $\tilde{s}$ indexed by $\mathcal{I}_n$. We have $y = D \tilde{s}$ where $D = [D_1^T, \ldots, D_N^T], y \triangleq [y_1^T, \ldots, y_N^T]^T$.

- $y$ is a permutation of $\tilde{s}$ and has block sparsity.
- The auxiliary variable $y$ is used to isolate $\tilde{s}$ from the group sparsity-inducing term $\sum \| \cdot \|_2$; the magnitude constraint is now imposed on $x$ instead of $\tilde{s}$. 

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The augmented Lagrangian can be written as

\[
L(\tilde{s}, y, x; \mu, \nu) = \frac{1}{2} \|\tilde{z} - \tilde{\Psi}\tilde{s}\|_2^2 + \nu^T (x - \tilde{s}) + \frac{\rho_2}{2} \|x - \tilde{s}\|_2^2 \\
+ \sum_{n=1}^{N} \left( \lambda \|y_n\|_2 + \mu_n^T (y_n - D_n\tilde{s}) + \frac{\rho_1}{2} \|y_n - D_n\tilde{s}\|_2^2 \right)
\]

(7)

where \( \rho_1, \rho_2 > 0 \) and \( \mu \triangleq [\mu_1^T, \ldots, \mu_N^T]^T \in \mathbb{R}^{2NM_tM_r} \) and \( \nu \in \mathbb{R}^{2NM_tM_r} \) are the Lagrangian multipliers.

ADMM is applicable if we group the variables into two blocks, i.e., \((y, x)\) and \(\tilde{s}\).

\[
(y^{k+1}, x^{k+1}) = \operatorname{arg\,min}_{y, x \in \Omega^{NM_tM_r}} L(\tilde{s}^k, y, x; \mu^k, \nu^k),
\]

\[
\tilde{s}^{k+1} = \operatorname{arg\,min}_{\tilde{s}} L(\tilde{s}, y^{k+1}, x^{k+1}; \mu^k, \nu^k),
\]

\[
\nu^{k+1} = \nu^k + \rho_2 (x^{k+1} - \tilde{s}^{k+1}),
\]

\[
\mu^{k+1} = \mu^k + \rho_1 (y^{k+1} - D\tilde{s}^{k+1}).
\]
The iterations for multipliers $\mu$ and $\nu$ are performed at cost $O(NM_t M_r)$.

The $y$-subproblem has computation cost $O(NM_t M_r)$

$$y_{n+1}^k = \max \left\{ \|\bar{s}_n^k\|_2 - \frac{\lambda}{\rho_1}, 0 \right\} \frac{\bar{s}_n^k}{\|\bar{s}_n^k\|_2}, \forall n \in \mathbb{N}_N^+, \quad (8)$$

where $\bar{s}_n^k = D_n \tilde{s}^k - \mu_n^k / \rho_1$. Recall that multiplying by $D_n$ only involves index selection.

The $x$-subproblem has computation cost $O(NM_t M_r)$

$$x_{n+1}^k = P_\Omega \left( \tilde{s}_{n+1}^k - \frac{\nu_n^k}{\rho_2} \right), \quad (9)$$

where $P_\Omega(x)$ projects $(x[i], x[i+NM_t M_r])$ onto the region $\{(x, y) | x^2 + y^2 \leq \omega_0\}$ for all $i \in \mathbb{N}_N^+$. Bo Li and Athina P. Petropulu

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For the $\tilde{s}$-subproblem, the minimum is achieved by

$$0 \in \frac{\partial}{\partial \tilde{s}} \mathcal{L}(\tilde{s}, y^{k+1}, x^{k+1}; \mu^k, \nu^k) = A\tilde{s} - b^k$$

(10)

where $A = \tilde{\Psi}^T \tilde{\Psi} + (\rho_1 + \rho_2)I_{2NM_tM_r}$ is block-diagonal and fixed in each iteration; $b^k = \tilde{\Psi}^T \tilde{z} + D^T \mu^k + \rho_1 D^T y^{k+1} + \nu^k + \rho_2 x^{k+1}$.

System (10) can be decomposed into a set of subsystems of equations, i.e.,

$$A_m \tilde{s}_{m}^{k+1} = b^k_m, \forall m \in \mathbb{N}^{+}_{2M_tM_r},$$

(11)

where $A_m = \begin{cases} 
\Psi_{ij}^T \Psi_{ij} + (\rho_1 + \rho_2)I_N & \text{if } m \in [1, M_tM_r] \\
A_{m-M_tM_r} & \text{otherwise}
\end{cases}$

with $j = \left\lfloor \frac{m-1}{M_t} \right\rfloor + 1$ and $i = m - (j - 1)M_t$.

$A_m$ is guaranteed to be strictly diagonal dominant and symmetric. The total number of operations to solve (11) is $O(N^2M_tM_r)$. 

The convergence of the above ADMM iterations is guaranteed by results in the ADMM literature.

The computational cost is low: $O(N^2 M_t M_r)$ v.s. $O((NM_t M_r)^3)$ for interior point based methods.

The estimation accuracy is improved by introducing the amplitude constraints.

The performance is robust over wide range of regularization parameter $\lambda$ (verified by the simulations).

The iterations of all variables exhibit separability.
Implementation Schemes and Discussions

- **Parallel Implementation**
  - all pairs \( (x^k[i], x^k[i + NM_t M_r]) \) in \( x^k \) are updated independent of others
  - a similar parallel scheme applies to \( \mu^k \) and \( \nu^k \), and the update of \( y^k_n \).

- **Fusion Center Aided Semi-Distributed Implementation**
  - \( x, s \) and \( \nu \) are divided into blocks, each of which can be updated locally at one receive node;
    The receive node \( j \) updates \( x_{m+1}^{k+1}, \nu_{m+1}^{k+1} \) and \( s_{m+1}^{k+1} \) for all \( m \in \mathcal{T}_j \triangleq \{(j - 1)M_t + i, M_t M_r + (j - 1)M_t + i | i \in \mathbb{N}_{M_t}^+\} \). The computation cost is \( O(N^2 M_t) \) at each node. \( v_{m+1}^{k+1} \in \mathbb{R}^N, m \in \mathbb{N}_{2M_t M_r}^+ \), denotes the \( m \)-th block of the uniformly partitioned vector \( v^{k+1} \).
  - A fusion center performs the update of \( y \) and \( \mu \);
    The computation cost is \( O(NM_t M_r) \) at the fusion center.
  - In each iteration, each receive node uploads \( \tilde{s}_{m+1}^{k+1} \) and downloads \( y_{m+1}^{k+1} \) from the fusion center.
    \( y_{m+1}^{k+1} \in \mathbb{R}^N \) denotes the \( m \)-th block of the uniformly partitioned \( D^T y^{k+1} \).
Simulations (1)

- We evaluate the performance of our proposed method using as metrics estimation error and running time.

Simulation setup
- 4 transmit and 4 receive nodes, waveforms with joint Gaussian entries;
- $\text{SNR} = 5 \text{dB}$;
- $25 \times 10$ grid points with $10m$ grid size;
- The magnitude of the complex reflection coefficients has uniform distribution $\mathcal{U}[0.1, 0.8]$. $\omega_0$ is chosen as 1.

Comparison methods
- BOMP [Gogineni, Nehorai 2011];
- GLasso using proximal gradient methods [Petropulu, Yu & Huang 2011];
- L-OPT with $\epsilon = 2\sqrt{LM_tM_r}\sigma_n$ [Li, Petropulu 2011] with knowledge of $\sigma_n$. 

Figure: Performance under different number of measurements. 10 targets; for GLasso $\lambda = 0.02$; and for the proposed method $\lambda = 2$, $\rho_1 = \rho_2 = 1$. 
Figure: Performance under different number of targets. 50 measurements; for GLasso $\lambda = 0.02$; and for the proposed method $\lambda = 2$, $\rho_1 = \rho_2 = 1$. 

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Figure: Performance under different values of $\lambda$. 
An ADMM-based efficient sparse signal recovery algorithm has been proposed for target estimation in distributed MIMO radar.

Simulation results have indicated that the proposed algorithm significantly lowers the computational complexity for target estimation and improves accuracy.

Parallel implementation has also been considered for further reduction of the execution time. A semi-distributed implementation, requiring a fusion enter with minimal computational power, has also been discussed.
Thank You
