On Wiretap Networks II

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***Abstract*— We consider the problem of securing a multicast network against a wiretapper that can intercept the packets on a limited number of arbitrary network links of his choice. We assume that the network implements network coding techniques to simultaneously deliver all the packets available at the source to all the destinations. We show how this problem can be looked at as a network generalization of the Ozarow-Wyner Wiretap Channel of type II. In particular, we show that network security can be achieved by using the Ozarow-Wyner approach of coset coding at the source on top of the implemented network code. This way, we quickly and transparently recover some of the results available in the literature on secure network coding for wiretapped networks. We also derive new bounds on the required secure code alphabet size and an algorithm for code construction.**

I. Introduction

Consider a communication network represented as a directed graph $G = (V, E)$ with unit capacity edges, an information source S that multicasts information to t receivers $R_1, \ldots, R_t$ located at distinct nodes. Assume that the min-cut value between the source and each receiver node is n. We know that a multicast rate of n is possible with linear network coding [1], [2]. We are here concerned with multicast networks in which there is an adversary that can access data on a certain number of links of his choice, and the goal is to maximize the multicast rate with the constraint of revealing no information about the multicast data to the adversary.

The problem of making a linear network code information theoretically secure in the presence of a wiretap adversary that can look at a bounded number, say μ, of network edges was first studied by Cai and Yeung in [3]. They considered directed graphs and demonstrated the existence of a code over an alphabet with at least $\binom{|E|}{\mu}$ elements which can support a secure multicast rate of up to $n-\mu$. They also showed that such codes can be designed in $\mathcal{O}(\binom{|E|}{\mu})$ steps. The required edge bandwidth and the secure code design complexity are main drawbacks of this pioneering work. Feldman *et al.* derived trade-offs between security, code alphabet size, and multicaat rate of secure linear network coding schemes in [4], by using ideas from secret sharing and abstracting network topology. Another approach was taken by Jain in [5] who obtained security by merely exploiting the topology of the network in question. Weakly secure network coding (which insures that only useless information rather than none is revealed to the adversary) was studied by Bhattad and Narayanan in [6], and practical schemes are missing in this case as well.

A related line of work considers a more powerful adversary, one that can also modify the packets he observes. Modifying a certain number of packets in networks which only route information simply results in their incorrect reception, whereas modifying the same number of packets carrying linear combinations of source packets can have a more harmful effect since it can result in incorrect decoding of all source packets. Such attacks are in network coding literature known as Byzantine modifications, and the Byzantine modification detection in networks implementing random network coding was studied by Ho *et al.* in [7] and Jaggi *et al.* in [8]. The approach they take is to introduce error correction coding at the source so that the packets carry not only data but also some redundant information derived from data which will help reduce the probability of incorrect decoding.

We also find coding at the source a natural approach to address the information theoretic security of wiretap networks. In a network where the min-cut value between the source and each receiver node is n and an adversary can access up to μ edges of his choice, we introduce at the source a coding scheme which ensures information theoretic security on the Ozarow-Wyner wiretap channel type II, introduced in [9] and [10], where the source transmits n symbols to the receiver and an adversary can access any μ of those symbols.

Ozarow and Wyner showed that the maximum number of symbols (say k) that the source can communicate to the receiver securely in the information theoretic sense is equal to $n-\mu$. They also showed how to encode the k source symbols into the n channel symbols for secure transmission. Clearly, if the n channel symbols are multicast over a network not performing coding (linear combining of the n symbols), the k source symbols remain secure in the presence of an adversary with access to any μ edges. We will illustrate later that this is is not necessarily the case when network coding is performed. However, we will show that a network code that preserves security of the k source symbols (coded into the n multicast symbols in the Ozarow-Wyner manner) can be designed over a sufficiently large field.

With the observations made by Feldman *et al.* in [4], it is easy to show that our scheme is actually equivalent to the one proposed in the pioneering work of Cai and Yeung in [3]. However, with our approach, we can quickly and transparently recover some of the results available in the literature on secure network coding for wiretapped networks. Since the publication of [3] in which the network code construction is based on the work of Li *et al.* in [2], a number of simpler network code construction algorithms have been proposed (see for example [11]), [12]. Computational complexity of network coding in terms of the number of coding nodes and ways to minimize

it have also been studied since then [12], [13], [14]. We will use these results to derive new bounds on the required secure code alphabet size and an algorithm for code construction.

This paper is organized as follows: In Sec. II, we briefly review the Ozarow-Wyner wiretap channel type II problem. In Sec. III, we introduce the network generalization of this problem. In Sec. IV, we present an algorithm for secure network code design and discuss the required code alphabet size. In Sec. V, we highlight some connections of this work with the previous work on secure network coding and more recent work on network error correction.

II. Wiretap Channel II

We first consider a point-to-point scenario in which the source can transmit n symbols to the receiver and an adversary can access any μ of those symbols [9], [10]. For this case, we know that the maximum number of symbols that the source can communicate to the receiver securely in the information theoretic sense is equal to $n-\mu$.

The problem is mathematically formulated as follows. Let $S=(s_1,s_2,\ldots,s_k)$ be the random variable associated with the k information symbols that the source wishes to send securely, $Y=(y_1,y_2,\ldots,y_n)$ the random variable associated with the symbols that are transmitted through the noiseless channel between the source and the receiver, and $Z=(z_1,z_2,\ldots,z_\mu)$ the random variable associated with the wire-tapped bits of Y. When $k\leq n-\mu$, there exists an encoding scheme that maps S into Y so that the uncertainty about S is not reduced by the knowledge of Z and S is completely determined (decodable) by the complete knowledge of Y, that is,

$$H(S|Z)=H(S) \text{ and } H(S|Y)=0. \tag{1}$$

For $n=2$, $k=1$, $\mu=1$, such a coding scheme can be organized as follows. If the source bit equals 0, then either 00 or 11 is transmitted through the channel with equal probability. Similarly, if the source bit equals 1, then either 01 or 10 is transmitted through the channel with equal probability.

	source bit s_1:	0	1
codeword y_1y_2 chosen at random from:		$\{00,11\}$	$\{01,10\}$

It is easy to see that knowledge of either y_1 or y_2 does not reduce the uncertainty about s_1, whereas the knowledge of both y_1 and y_2 is sufficient to completely determine s_1, namely, $s_1=y_1+y_2$.

In general, $k=n-\mu$ symbols can be transmitted securely by a coding scheme based on an $[n,n-k]$ linear MDS code $\mathcal{C}\subset\mathbb{F}_q^n$. In this scheme, the encoder is a probabilistic device which operates on the space $\mathbb{F}_q^n$, where q is a large enough prime power, partitioned into q^k cosets of $\mathcal{C}$. The k information symbols are taken as the syndrome which specifies a coset, and the transmitted word is chosen uniformly at random from the specified coset. The decoder recovers the information symbols by simply computing the syndrome of the received word. Because of the properties of MDS codes, knowledge any $\mu=n-k$ or fewer symbols will leave uncertainty of the k information symbols unchanged. The code used in the above example is the $[2,1]$ repetition with the parity check matrix

$$H=\begin{bmatrix}1 & 1\end{bmatrix}. \tag{2}$$

III. Wiretap Network II

We now consider again an acyclic multicast network $G=(V,E)$ with unit capacity edges, an information source, t receivers, and the value of the mincut to each receiver equal to n. The goal is to maximize the multicast rate with the constraint of revealing no information about the multicast data to the adversary that can access data on any μ links. We assume that the adversary knows the implemented network code, *i.e.* all the coefficients of the linear combinations that determine the packets on each edge. Moreover, the adversary is aware of any shared randomness between the source and the destinations. The last assumption rules out the use of traditional "key" cryptography to achieve security.

We know that a multicast rate of n is possible with linear network coding [1], [2]. It is interesting to ask whether, using the same network code, the source can multicast $k\leq n-\mu$ symbols securely if it first applies a secure wiretap channel code (as described above) mapping k into n symbols. Naturally, this would be a solution if a multicast rate of n can be achieved just by routing.

Consider this approach for the butterfly network shown in Fig. 1 where we have $n=2$, $k=1$, $\mu=1$. If the source applies the coding scheme described in the previous section and the usual network code as in Fig. 1-a, the adversary will be able to immediately learn the source bit if he taps into any of the edges BE, EF, ED. Therefore, a network code can brake down a secure wiretap channel code. However, if the network code is changed so that node B combines its inputs over *e.g.*, $\mathbb{F}_3$ and the BE coding vector is $\begin{bmatrix}1 & \alpha\end{bmatrix}$ where α is a primitive element of $\mathbb{F}_3$ (as in Fig. 1-b), the wiretap channel code remains secure, that is, the adversary cannot gain any information by accessing any single link in the network. Note that the wiretap channel code based on the MDS code with $H=\begin{bmatrix}1 & 1\end{bmatrix}$ remains secure with any network code whose BE coding vector is linearly independent of $\begin{bmatrix}1 & 1\end{bmatrix}$.

We will next show that the source can multicast $k\leq n-\mu$ symbols securely if it first applies a secure wiretap channel code based on an MDS code with a $k\times n$ parity check matrix H if the network code is such that no linear combination of $\mu=n-k$ or fewer coding vectors belongs to the space spanned by the rows of H. Let $W\subset E$ denote the set of $|W|=\mu$ edges the wiretapper chooses to observe, and $Z_W=(z_1,z_2,\ldots,z_\mu)$ the random variable associated with the packets carried by the edges in W. Let C_W denote the matrix whose rows are the coding vectors associated with the observed edges in W. As in the case of wiretap channel, $S=(s_1,s_2,\ldots,s_k)$ denotes the random variable associated with the k information symbols that the source wishes to send securely, and $Y=(y_1,y_2,\ldots,y_n)$ the random variable associated the n wiretap channel code symbols. The n symbols of Y will be multicast through the network by using linear network coding. Consider $H(S,Y,Z_W)$ with the security requirement $H(S|Z_W)=H(S)$ for all $W\subset E$:

$$\begin{aligned}
&\underbrace{H(S|Z_W)}_{=H(S)}+H(Y|SZ_W)=H(Y|Z_W)+\underbrace{H(S|YZ_W)}_{=0}\\
&\Rightarrow H(Y|SZ_W)=H(Y|Z_W)-H(S)\\
&\Rightarrow 0\leq n-\text{rank}(\mathbf{C}_W)-k
\end{aligned}$$

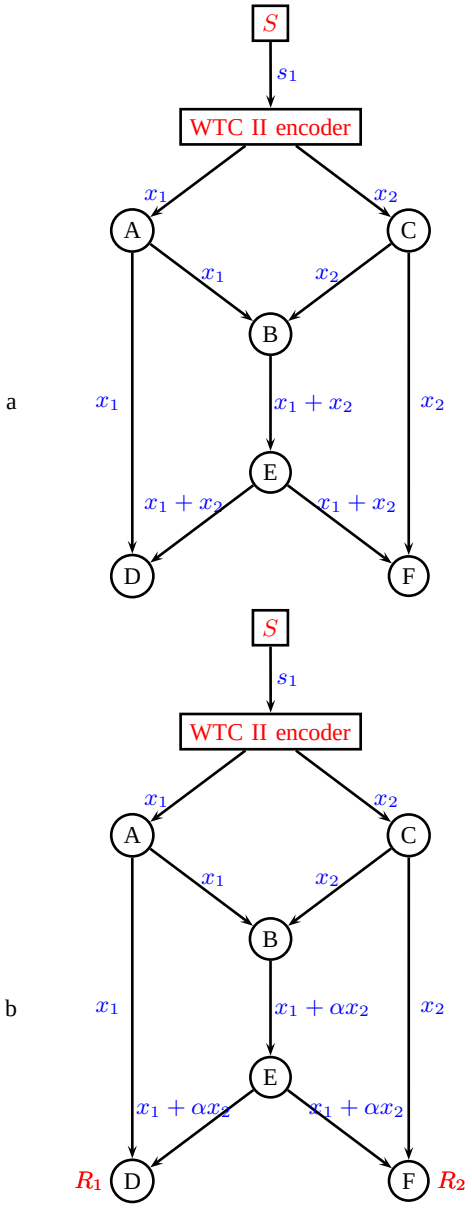

Fig. 1. Single-edge wiretap butterfly network with a) insecure network code and b) secure network code.

Since there is a choice of edges such that $\text{rank}(C_W) = \mu$, the maximum rate for secure transmission is bounded as

$$k \leq n - \mu.$$

If the bound is achieved with equality, we have $H(Y|SZ_W) = 0$ and consequently, the system of equations

$$\begin{bmatrix} S \\ Z_w \end{bmatrix} = \begin{bmatrix} H \\ C_W \end{bmatrix} \cdot Y$$

has to have unique solution for all W for which $\text{rank}(C_W) = \mu$. That is,

$$\text{rank} \begin{bmatrix} H \\ C_W \end{bmatrix} = n \quad \text{for all } C_W \text{ s.t. } \text{rank}(C_W) = \mu. \qquad (3)$$

This analysis essentially proves the following result:

Theorem 1: Let $G = (V, E)$ be an acyclic multicast network with unit capacity edges, an information source and the mincut value to each receiver equal to n. A wiretap code at the source based on an MDS code with a $k \times n$ parity check matrix H and a network code such that no linear combination of $\mu = n - k$ or fewer coding vectors belongs to the space spanned by the rows of H make the network information theoretically secure against a wiretap adversary who can observe at most $\mu \leq n - k$ edges. Any adversary able to observe more than $n - k$ edges will have uncertainty about the source smaller than k.

The above analysis shows that the maximum throughput can be achieved by applying a wiretap channel code at the source and then designing the network code while respecting certain constraints. The decoding of secure source symbols S is then merely matrix multiplication of the decoded multicast symbols Y. The method gives us a better insight of how much information the adversary gets if he can access more edges than the code is designed for. It also gives us an insight on how to simply design secure network codes in some cases over much smaller alphabets then currently deemed necessary. Both claims are illustrated in the example below.

IV. Network Code Design Alphabet Size

The approach described previously in the literature for finding a secure multicast network code consisted of decoupling the problem of designing a multicast network code and making it secure by using some code on top of it. Feldman *et al.* showed in [4] that there exist networks where the above construction might require a quite large field size. We investigate here a different construction that, as was hinted in the conclusion of [4], exploits the topology of the network. This is accomplished by incorporating the security constraints in the *Linear Information Flow* (LIF) algorithm of [11] that constructs linear multicast network codes in polynomial time in the number of edges in the graph. The result is a better lower bound on the sufficient field size. However, the modified LIF algorithm does not have polynomial time complexity.

We start by giving a brief high level overview of the LIF algorithm of [11]. The inputs of the algorithm are the network, the source node, the t destination nodes and the number n of packets that need to be multicast to all the destinations. Assuming the min-cut between the source and any destination is at least n, the algorithm outputs a linear network code that guaranties the delivery of the n packets to all the destinations.

The algorithm starts by 1) finding t flows $F_1, F_2, \ldots, F_t$ of value n each, from the source to to each destination and 2) setting t $n \times n$ matrices B_{F_j} (one for each receiver) equal to $I_{n\times n}$ Then, it goes over the network edges, visiting each one in topological order. In each iteration, the algorithm finds a suitable local encoding vector for the visited edge, and updates the t matrices B_{F_j}, each formed by the global encoding vectors of the n last visited edges in the flow F_j. The algorithm maintains the invariant that the matrices B_{F_j} remain invertible

after each iteration. Thus, when it terminates, each destination will get n linear combination of the original packets that form a full rank system. Thus each destination can solve for these packets by inverting the corresponding system.

An important result of the previous algorithm, is that a field of size at least t (the number of destinations) is always sufficient for finding the desired network code. As shown in [11, Lemma 8], this follows from the fact that a field of size larger or equal to t is actually sufficient for satisfying the condition that the t matrices B_{F_j} are always invertible.

We modify the LIF algorithm so it outputs a secure network code in the following way. We fix the $k \times n$ parity check matrix H. WLOG, we assume that the μ packets observed by the wiretapper are linearly independent, *i.e.* rank $C_W = \mu$. We denote by e_i the edge visited at the i-th iteration of the LIF algorithm, and by P_i the set of the edges that have been processed by the end of it. Then, we extend the set of invariants to make sure that the encoding vectors are chosen so the matrices $M_W = \begin{bmatrix} H \\ C_w \end{bmatrix}$ are also invertible; which by Theorem 1 achieves the security condition. More precisely, using the same techniques as the original LIF algorithm, we make sure that by the end of the ith iteration, the matrices B_{F_j} and the matrices M_{W_i} are invertible; where $W_i = \{e_i\} \cup W'$ and W' is a subset of P_i of order $\mu - 1 = n - k - 1$. The total number of the matrices that need to be kept invertible in this modified version of the LIF algorithm is at most $\binom{|E|-1}{\mu-1} + t$ (which corresponds to the last iteration). Thus, similarly as in [11, Lemma 8], we obtain the following improved bound on the alphabet size for secure multicast:

Theorem 2: Let $G = (V, E)$ be an acyclic network with unit capacity edges, an information source, and the mincut value to each of the t receivers equal to n. A secure mulitcast at rate $k \leq n$ in the presence of a wiretapper who can observe at most $\mu \leq n - k$ edges is always possible over the alphabet $\mathbb{F}_q$ of size

$$q > \binom{|E| - 1}{\mu - 1} + t. \tag{4}$$

Bound (4) can be further improved by realizing as was first done in [12] that not all edges in the network carry different linear combination of source symbols. Langberg *et al.* showed in [13, Thm. 5] that the problem of finding multicast network codes for a network G can be reduced to solving the same problem for a special equivalent network $\widehat{G}$ with same parameters n and t, which has the properties that all nodes except the source and the destinations have total degree 3 and at most n^3t^2 of its nodes have in-degree 2. These nodes are called *encoding nodes*, whereas the other ones are called *forwarding nodes* since the packets carried by their outgoing edges are just copies of the ones available at their single incoming edge. Given a network code for $\widehat{G}$, a one for G can be found efficiently over the same field. And, the set of global encoding vectors of the edges of G would be a subset of the one of $\widehat{G}$.

Going back the security problem over a network G, one can try to find a secure network code for the equivalent network $\widehat{G}$, and then use the procedure described in [13] and [14] to construct a network code for G which will also be secure. Now consider the problem of finding secure network codes for $\widehat{G}$. This problem will not change if the wiretapper is not allowed to wiretap the *forwarding edges*. Therefore, the set of edges that the wiretapper might have access to consists of the encoding edges and the edges outgoing from the source, and is of order $n^3t^2+\delta$, where δ is the out-degree of the source. Now, applying Theorem 2 on $\widehat{G}$ and taking into consideration the restriction on the edges that can be potentially wiretapped, we obtain the following bound on the sufficient field size which is independent of the size of the network.

Corollary 1: For the transmission scenario of Thm. 2, a secure mulitcast network code always exists over the alphabet $\mathbb{F}_q$ of size

$$q > \binom{k^3t^2 + \delta}{\mu - 1} + t. \tag{5}$$

For networks with two sources, we can completely settle the question on the required alphabet size for a secure network code. Note that the adversary has to be limited to observing at most one edge of his choice. Based on the work of Fragouli and Soljanin in [12], the coding problem for these networks is equivalent to a vertex coloring problem of some specially designed graphs, where the colors are actually the points on the projective line $\mathbb{PG}(1, q)$:

$$[0\,1],\ [1\,0],\ \text{and}\ [1\,\alpha^i]\ \text{for}\ 0 \leq i \leq q-2, \tag{6}$$

where α is a primitive element of $\mathbb{F}_q$. Clearly, any network with two sources and arbitrary number of receives can be securely coded by reducing the set of available colors in (6) by removing point (color) $[1\,1]$ and applying a wiretap code based on the matrix $H = [1\,1]$ as in the example above. Alphabet size sufficient to securely code all network with two sources also follows from [12]:

Theorem 3: For any configuration with two sources t receivers, the code alphabet $\mathbb{F}_q$ of size

$$\lfloor \sqrt{2t - 7/4} + 1/2 \rfloor + 1$$

is sufficient for a secure network code. There exist configurations for which it is necessary.

The wiretap approach to network security also provides the exact alphabet size and secure code for a class of networks known as combination networks and are illustrated in Fig. 2. There are $\binom{M}{n}$ receiver nodes. Note that each n nodes of the

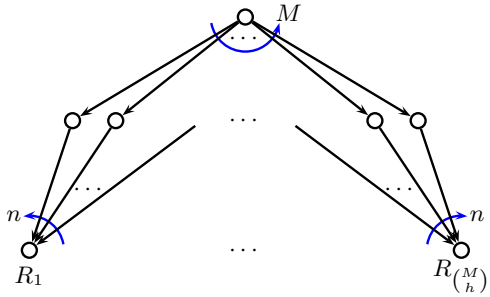

Fig. 2. Combination $B(n, M)$ network.

second layer are observed by a receiver. It is easy to see that an $[M + k, n]$ Reed Solomon code can be used, namely, the

first k rows its parity check matrix can be used for the cosset code and the rest as the coding vectors of the M edges going out of the source.

V. Connections with Other Schemes

A number of connections between secure network coding with the concurrent work on network error correction can be observed [15], [16], [17]. We here describe the relationship between the proposed scheme and previously known constructions. Cai and Yeung were first to study the design of secure network codes for multicast demands [3]. They showed that, in the setting described above, a secure network code can be found for any $k \leq n - \mu$. Their construction is equivalent to the following scheme:

1) Generate a vector $R = (r_1, r_2, \ldots, r_\mu)^T$ choosing its components uniformly at random over $\mathbb{F}_q$,
2) Form vector X by concatenating the μ random symbols R to the k source symbols S:
$$X = \begin{bmatrix} S \\ R \end{bmatrix} = (s_1, \ldots, s_k, r_1, \ldots, r_\mu)^T$$
3) Chose an *invertible* $n \times n$ matrix over $\mathbb{F}_q$ and a linear code multicast (LCM) [2] to ensure the security condition (1). (It is shown in [3, Thm. 1] that such LCM and T can be found provided that $q > \binom{|E|}{\mu}$.)
4) Compute $Y = TX$ and multicast Y to all the destinations by using the constructed code.

Feldman *et al.* considered also the same problem in [4]. Adopting the same approach of [3], they showed that in order for the code to be secure, the matrix T should satisfy certain conditions ([4, Thm. 6]), that we restate here for convenience: In the above transmission scheme, the security condition (1) holds if and only if any set of vectors consisting of

1) at most μ linearly independent global edge coding vectors and/or
2) any number of vectors from the first k rows of T^{-1}

is linearly independent. They also showed that if one sacrifices in the number of information packets, that is, take $k < n - \mu$, then one can find secure network codes over fields of size much smaller than the very large bound $q > \binom{|E|}{\mu}$.

We will now show that our approach based on coding for the wiretap channel at the source is equivalent to the above stated scheme [3] with the conditions of [4].

Claim 1: Let T and $\mathcal{C}$ be a matrix and a corresponding secure network code satisfying the above conditions. Set $H = T^*$ where T^* is the $k \times n$ matrix formed by taking the first k rows of T^{-1}. Then H and $\mathcal{C}$ satisfy the condition of Thm. 1.

Proof: Consider the secure multicast scheme of [3] as presented above. For a given information vector $S \in \mathbb{F}_q^k$, let $B(S)$ be the set of all possible vectors $Y \in \mathbb{F}_q^n$ that could be multicast through the network under this scheme. More precisely,

$$B(S) = \left\{ Y \in \mathbb{F}_q^n | Y = TX, X = \begin{bmatrix} S \\ R \end{bmatrix}, R \in \mathbb{F}_q^{n-k} \right\}.$$

Then, for all $Y \in B(S)$, we have $T^*Y = T^*T \begin{bmatrix} S \\ T \end{bmatrix} = S$. Therefore, any $Y \in B(S)$ also belongs to the coset of the space spanned by the rows of T^* whose syndrome is equal to S. Moreover, since T is invertible, $|B(S)| = 2^{n-k}$ implying that set $B(S)$ is exactly that coset. The conditions of [4] as stated above directly translate into (3), the remaining condition of Thm. 1. ∎

VI. Conclusion

We considered the problem of securing a multicast network implementing network coding against a wiretapper capable of observing a limited number of links of his choice, as defined initially by Cai and Yeung. We showed that the problem can be formulated as a generalization of the wiretap channel of type II (which was introduced and studied by Ozarow and Wyner), and decomposed into two sub-problems: the first one of designing a secure wiretap channel code and the second of designing a network code satisfying some additional constraints. We proved there is no penalty to pay by adopting this separation, which we find in many ways illuminative.

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