Index Coding, Caching & Distributed Storage

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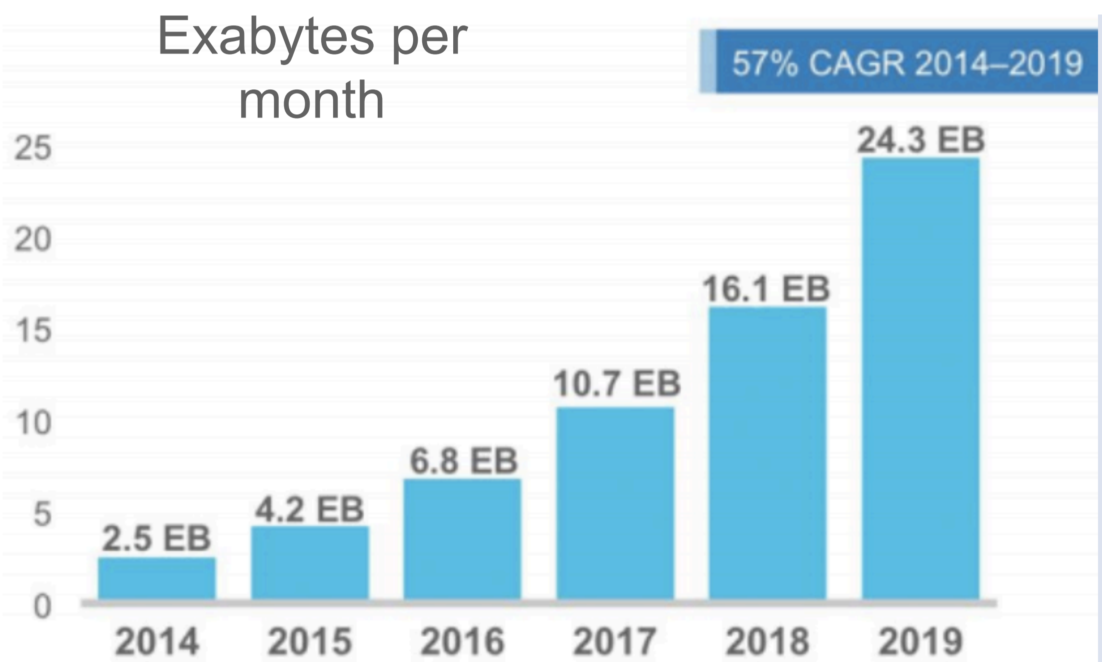
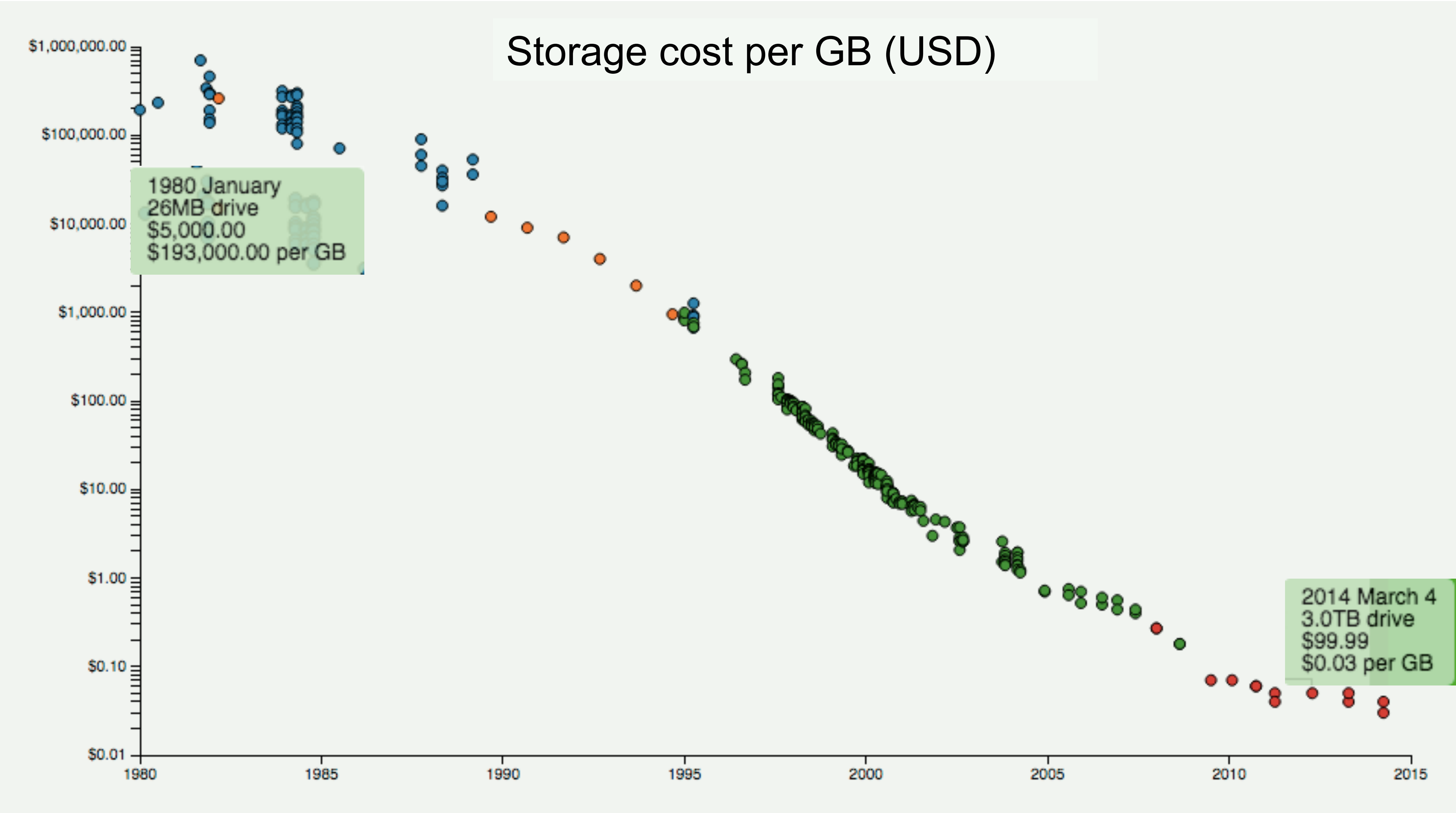
Big Data vs. Wireless



Cisco forcast

$$\infty \equiv 5 \text{ GB} \pmod{\mathbf{at\&t}}$$

Meanwhile, Storage is Getting Very Cheap



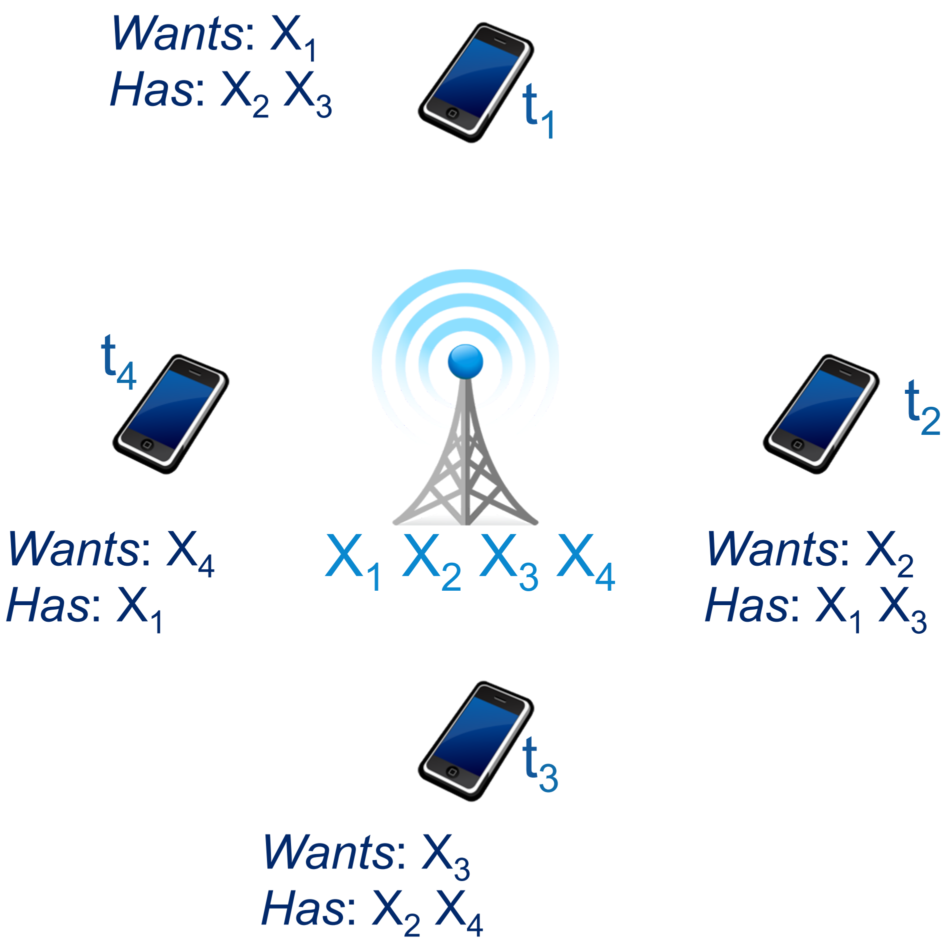
http://www.mkomo.com/cost-per-gigabyte-update

How Can Storage Help?

- Index coding: [Birk & Kol ’98] + …
- COPE: [Katti et al. ‘06]+…
- Caching: [Maddah-Ali & Niesen ’13] + …
- Femto-caching: [Golrezai et al. ’12]….
- ….

Content is cached (stored) on mobile devices during off-peak hours

Index Coding Example



[Birk & Kol '98]

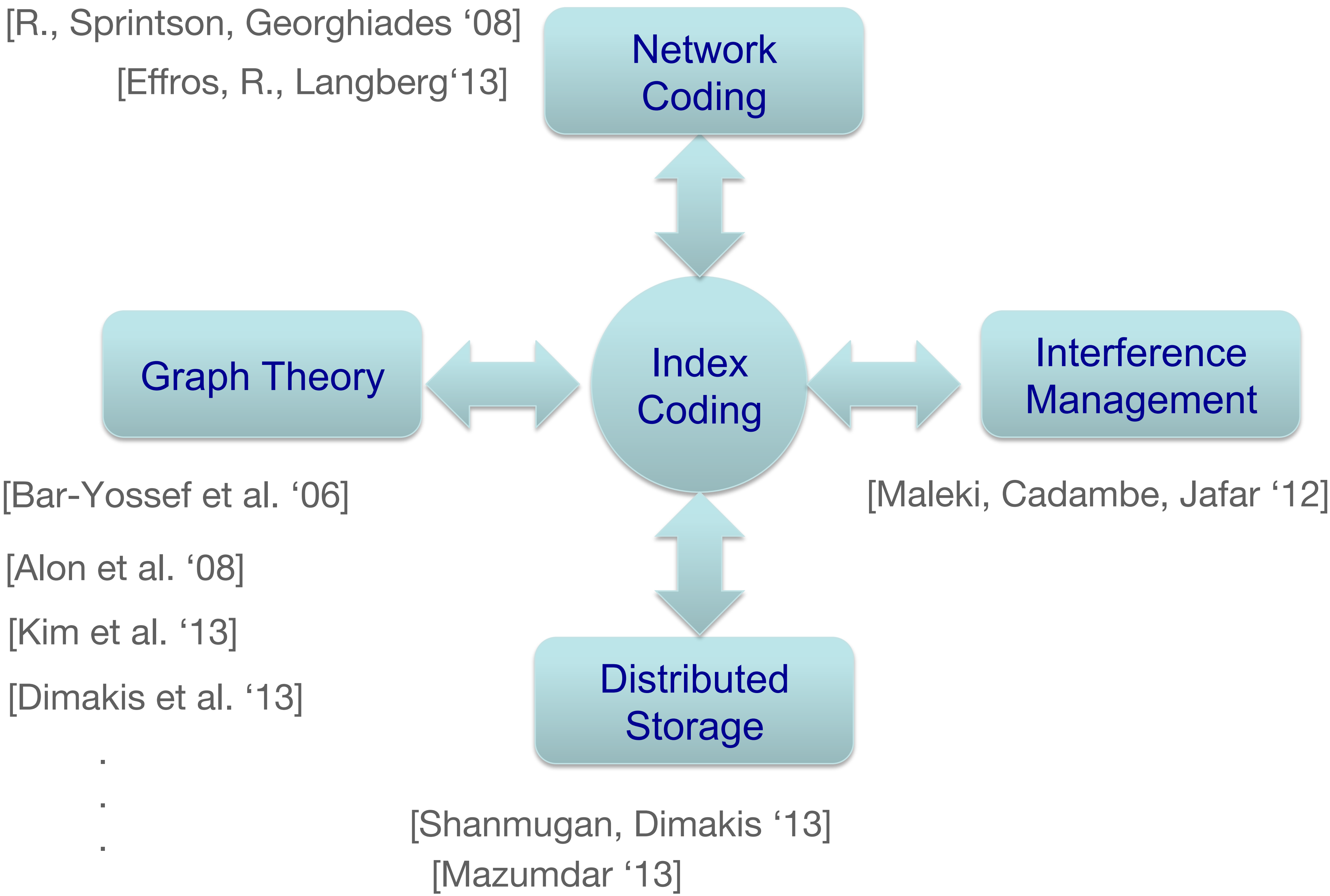
Trans-mission #	Index code 1	Index code2
1	X_1	X_1+X_2
2	X_2	X_3
3	X_3	X_4
4	X_4	

$L=4$ $L=3$

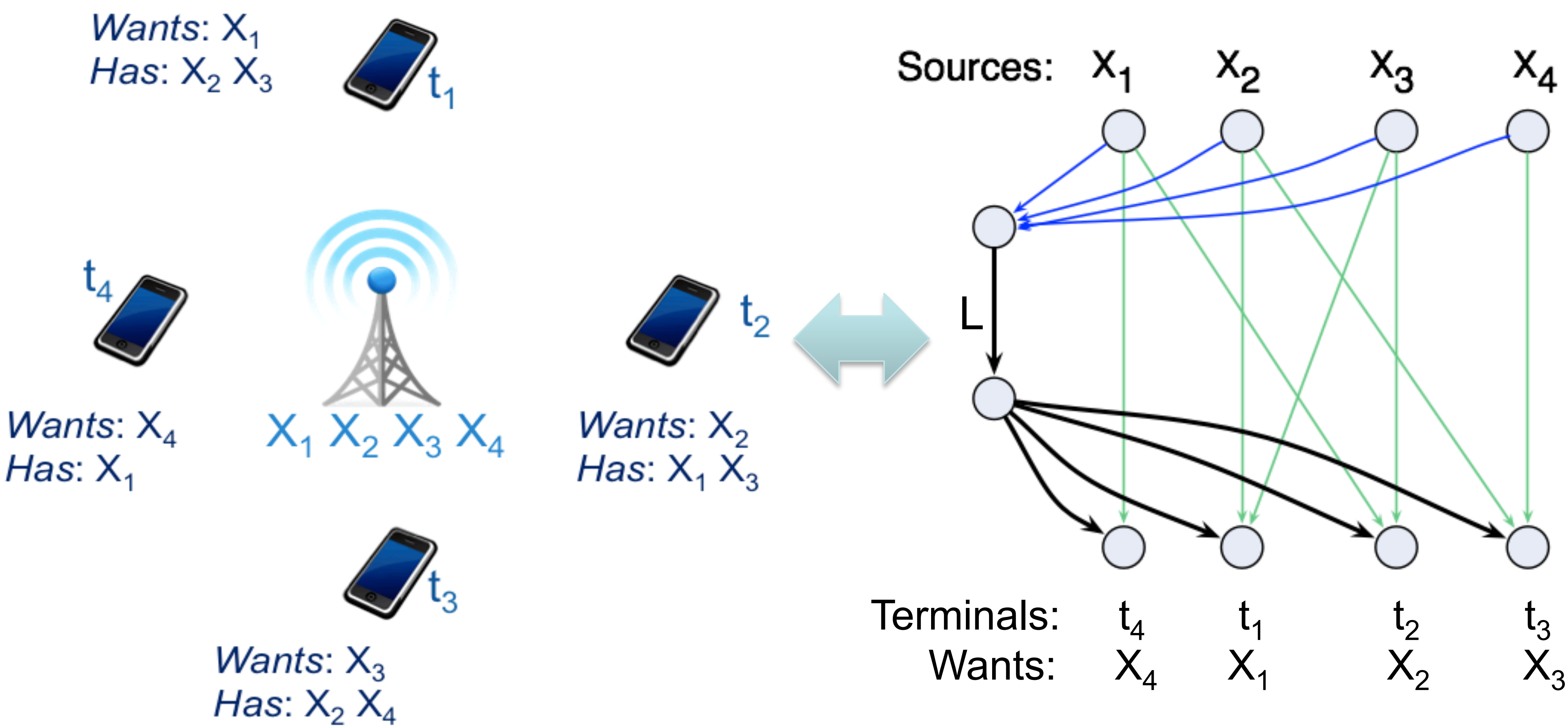
Can we do better?

- Cached data is NOT a design parameter now
- Even randomly cached data independent of the demands can help

Connections to Other Fields



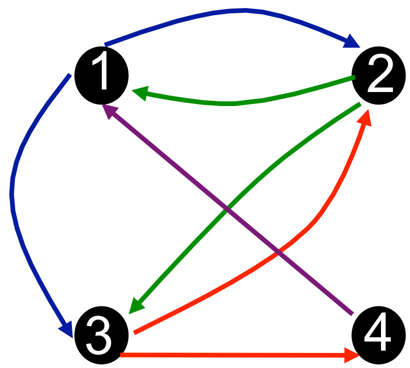
Equivalence to Network Coding



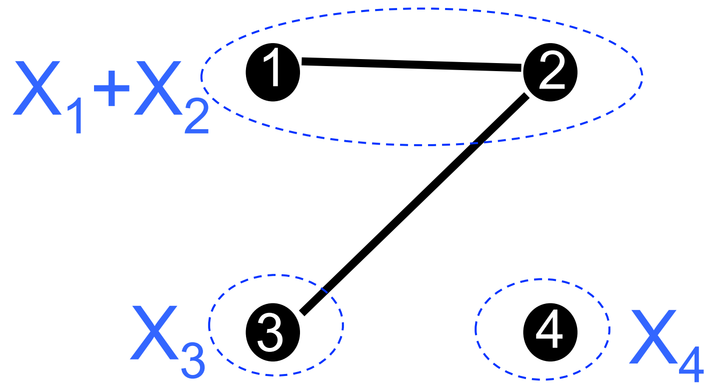
Theorem: [R,Sprintson, Georghiades'08] [Effros,R,Langberg ISIT'13]

Given any network coding problem, one can construct an index coding problem and an integer L such that given any network code, one can efficiently construct a index code of length L, and vice versa.

Index Coding & Graph Coloring



Side info graph G_d



Chromatic nbr of $\bar{G}$

Independence nbr

$$\alpha(G_d) \leq c(G_d) \leq L^*_{min} \leq \chi_f(\bar{G}) \leq \chi(\bar{G})$$

Shannon capacity
[Haemers '79]

Fractional Chromatic nbr
[Blasiak, Kleinberg, Lubetzky '11]

$$\leq \chi_{f\ell}(G)$$

Fractional local chrom. nbr
[Shanmugam, Dimakis, Langberg'13]

Index Coding & Rank Minimization

	X_1	X_2	X_3	X_4
t_1	1	*	*	0
t_2	*	1	*	0
t_3	0	*	1	*
t_4	*	0	0	1

- Linear case: $L^*_{min} = \min rk(M)$ [Bar-Yossef et al. '06]
- Computing L^*_{min} is NP hard. [Rouayheb. et al. '07] [Peeters '96]
- Methods for constructing index codes from rank minimization [Esfahanizadeh, F. Lahouti, and B. Hassibi '14] [Huang, R '15]

Index Coding & Rank Minimization

$$
\begin{array}{cc|c|c|c|c|}
 & & X_1 & X_2 & X_3 & X_4 \\
\hline
X_1 & t_1 & 1 & 0 & 0 & 0 \\
\hline
X_2 & t_2 & 1 & 1 & 0 & 0 \\
\hline
X_3 & t_3 & 0 & 0 & 1 & 0 \\
\hline
X_4 & t_4 & 0 & 0 & 0 & 1 \\
\hline
\end{array}
$$

Matrix M

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- Computing L^*_{min} is NP hard. [Rouayheb. et al. '07] [Peeters '96]
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[Esfahanizadeh, F. Lahouti, and B. Hassibi '14] [Huang, R '15]

Index Coding & Rank Minimization

$X_1+X_2+X_3$

X_1+X_4

	X_1	X_2	X_3	X_4
t_1	1	1	1	0
t_2	1	1	1	0
t_3	0	1	1	1
t_4	1	0	0	1

Matrix M

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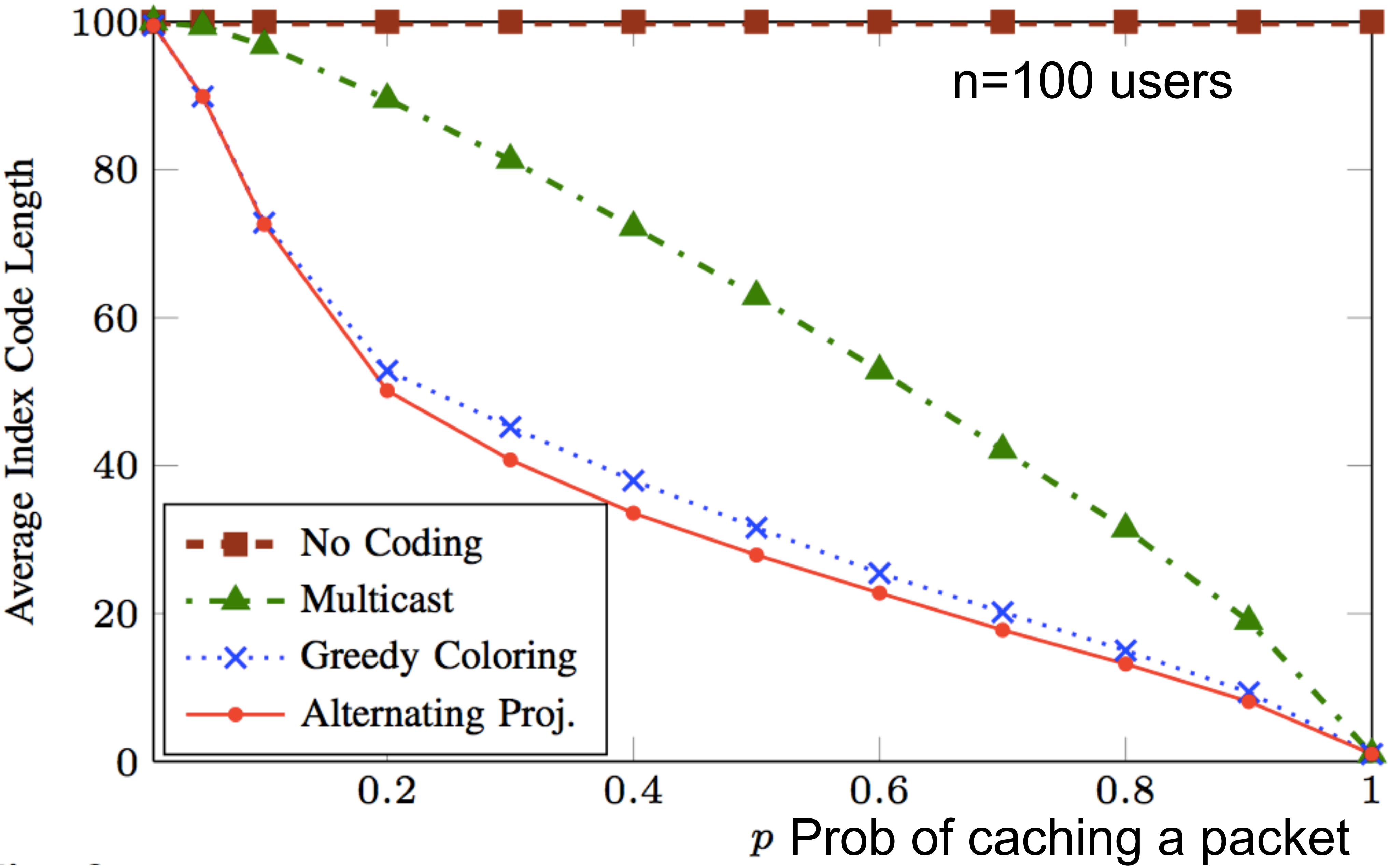
Performance of different methods

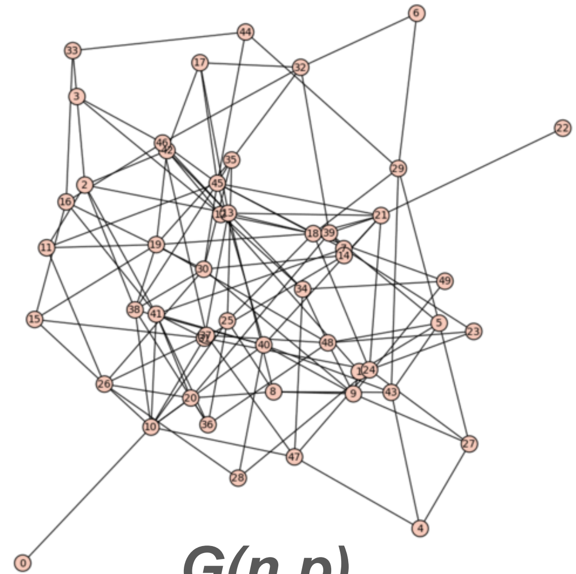


[Huang, R.'15]

Index Coding on Erdős-Rényi Graphs

Independence nbr Chromatic nbr

$$\alpha(G) \leq L^*_{min} \leq \chi(\bar{G})$$



G(n,p)

- When $n \to \infty$, we have with prob 1

$$\log n \leq L^*_{min} \leq \frac{n}{\log n}$$

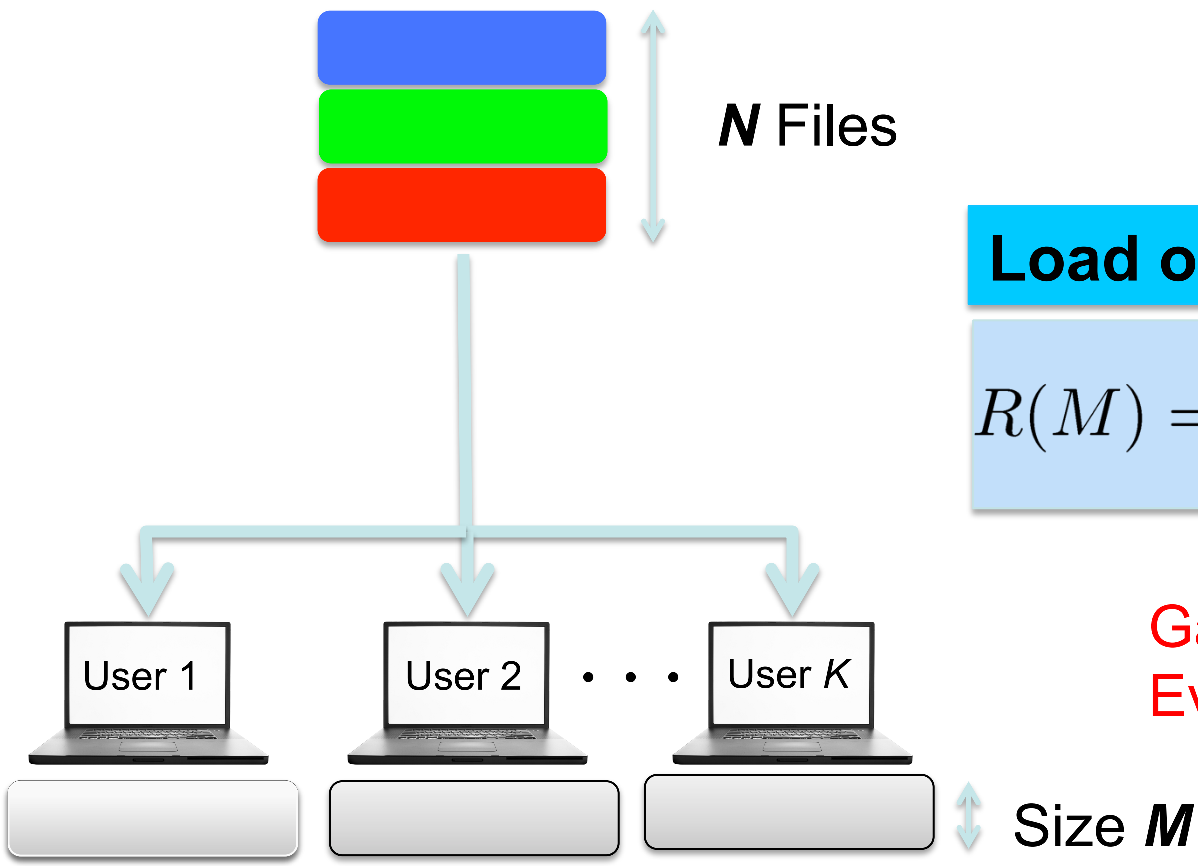
- Can improve the lower bound [Haviv & Langberg '11]

$$c\sqrt{n} \leq L^*_{min} \leq \frac{n}{\log n}$$

- Coloring is the best upper bound we know on random graphs. Is it tight? OPEN

Caching Networks

Maddah-Ali, Niesen, 2012



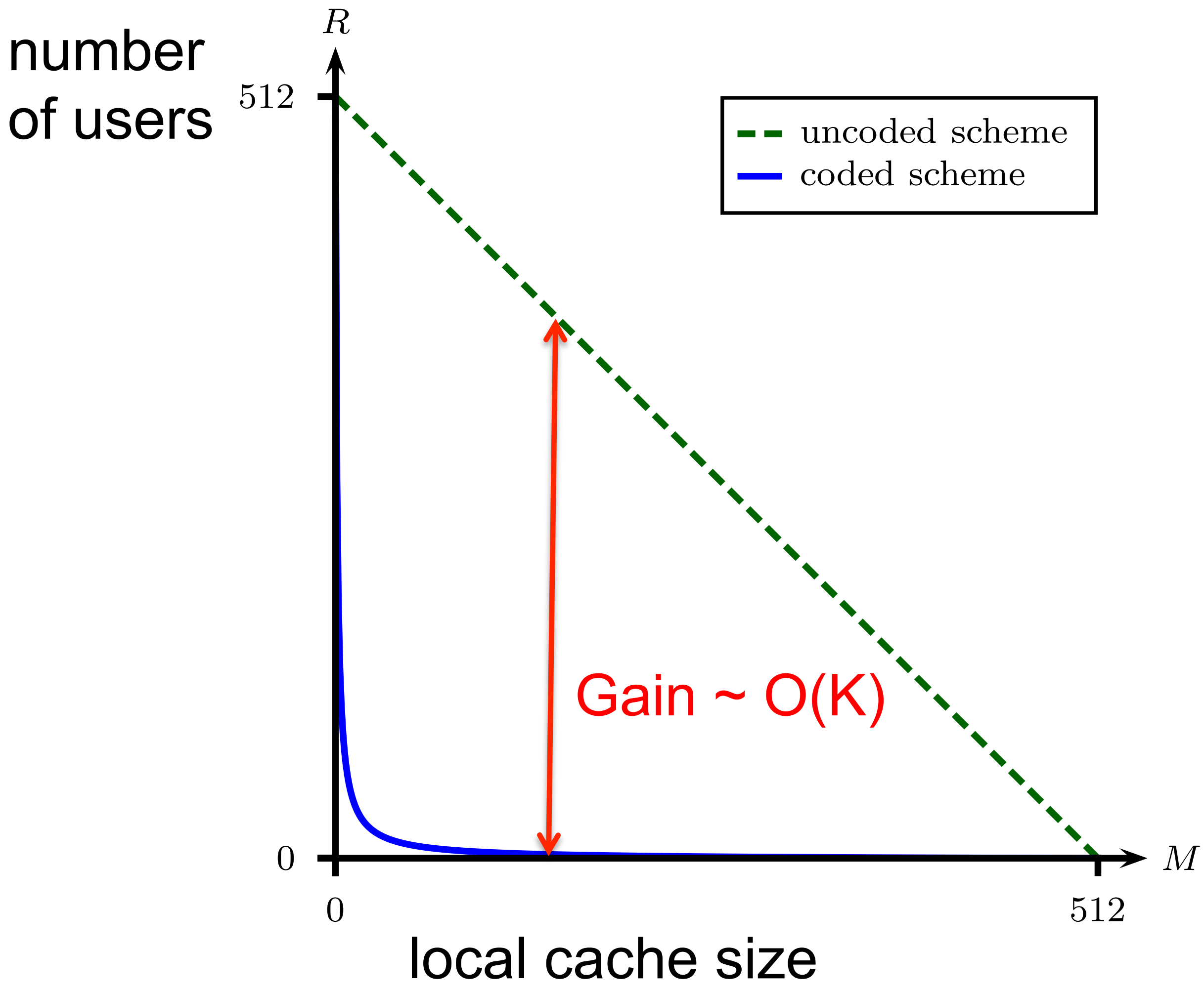
Load of the link

$$R(M) = K\left(1 - \frac{M}{N}\right)\frac{1}{1+KM/N}$$

Gain from entire caches (*KM*),
Even-though caches are isolated

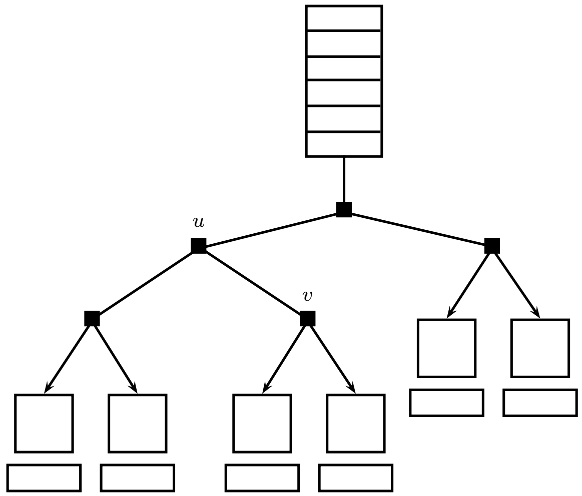
- **Placement phase:** Populate caches (prefetching), Design the caches
  - Demands are not known yet
- **Delivery phase:** Reveal request, deliver content
  - Minimize the rate in the bottleneck

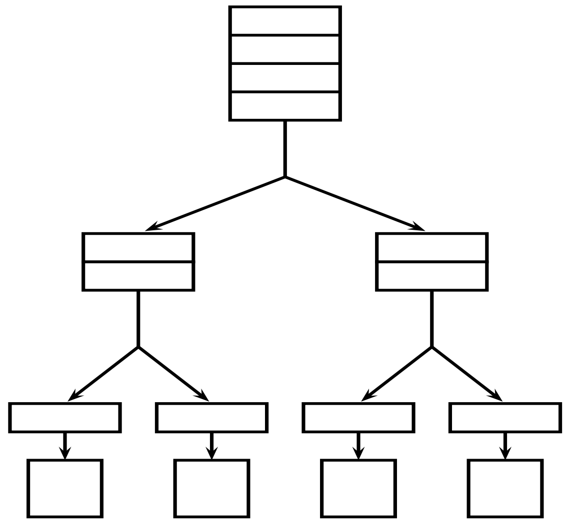
Caching Networks

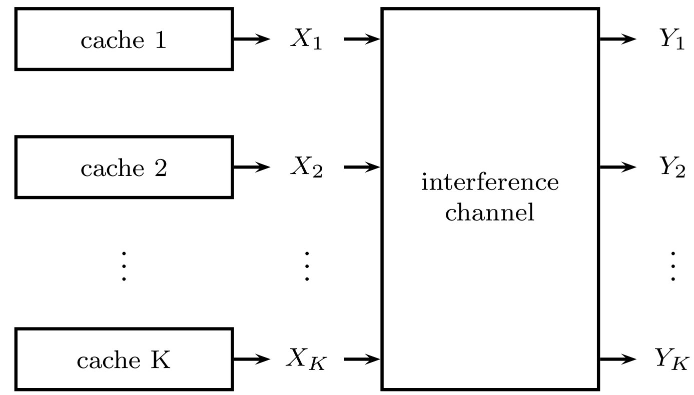


Caching Networks

Topologies







Scenarios

- Non-Uniform demands
- Online Cache updating
- Delay-Limited Delivery
- Complexity-Limited (Alex Plenary Talk)
- Improving Approximation (Ravi's Talk)
- Secure Delivery

Research Teams:

Maddah-Ali, Niesen, Pedarsani [2012-15]
Dimakis, Molisch, Caire, Shanmugam, Golrezaei, [2012-15]
Avestimehr. Naderializadeh,[2014]
Sengupta, Tandon and Clancy [2014-15]
Shanmugam, Ji, Tulino, Llorca, Dimakis [2014-2015]
Diggavi, Hachem, Karamchandani, Maddah, Niesen [2013-2015]
Tian [2015]

QUESTIONS?